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A PROPOSAL FOR A MODEL OF THE GROWTH OF FOREST FIRES

WORKING NOTE 6/74

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Summary and Conclusions

This note describes a proposed model which may be used to represent the relationship between the growth of a fought fire and the number of jets which are applied. The model can be used to predict the effect of applying jets at different times and hence can be used in the study of the number of appliances required at each fire station.

The model is derived by considering simple mathematical forms of the fire growth curve which can be used to describe the observed behaviour of real fires and which are compatible with the observed characteristics of experimental fires. Two different forms of a quadratic growth model are derived and some estimates of the coefficients are given.

The model described here is deterministic and it remains to investigate the properties of an equivalent stochastic model and to find some means of validating or disproving this proposed model.

A PROPOSED MODEL FOR THE GROWTH OF A FOUGHT FIRE

Contents:-

THE GROWTH OF A FOUGHT FIRE

1. INTRODUCTION

In a study of the number of pumps per station or a study of the first attendance rules it is essential to have an understanding of the way in which fires grow after the arrival of the first pump. This working note is a description of a proposed model.

This model is applicable to building fires which have spread beyond the room of origin.

2. OBSERVATIONS FROM THE DATA

Any model which represents the growth of a fought fire must be capable of "explaining" the following effects which are observed in the SAF2 data.

i. The average amount of spread for fires which are of the same size at arrival increases with the number of jets used. This is due to the differing "fierceness" of the fires. An examination of the number of jets used against the final area also shows that spreading fires use more jets.

ii. The proportion of fires which spread after arrival increases with the size of fire at arrival.

iii. But the average amount of spread (for spreading and non-spreading fires) is approximately constant for all sizes of fire.

iv. ii and iii imply that for fires which spread, the average spread ($^{A_F}/_{A_0}$) is less for large fires.

v. Fires for which all the pumps arrive simulataneously are still likely to spread after arrival.

vi. The control time (as recorded on the K433 forms) increases with increasing final area.

3. REGRESSION ANALYSIS

An analysis of the relationship between the amount of spread and the times at

which the jets were applied would provide the simplest possible model of fire growth. A regression analysis was tried using a model which was based on the following assumptions.

(i) The rate of growth of fires is exponential.

(ii) Each pump supplied one jet.

(iii) The arrival times of the pumps are the times at which the jets were applied.

The model used to represent the growth of a j jet fire was: $A_F = A_o \; e^k \; e^{\beta_1 t_1} \; e^{\beta_2 t_2} \; \ldots\ldots\ldots \; e^{\beta_j t_j}$

Where A_F is the area damaged at extinction

A_o is the area damaged at arrival

β is the exponential growth rate

t_{i-1} is the $i-1^{th}$ interarrival time ie the interval between the arrival of the i^{th} and $(i-1)^{th}$ pump

e^k represents any growth not accounted for by "interarrival" growth.

This regression analysis was tried on multi-storey non-dwelling fires which had spread beyond the room of origin.

1 jet fires (n = 598)	$A_F/A_o = 1.14$	Mean $A_F/A_o = 1.14$
2 jet fires (n = 366)	$A_F/A_o = 1.16 \; e^{0.t_1}$	Mean $A_F/A_o = 1.16$
3 jet fires (n = 146)	$A_F/A_o = 1.16 \; e^{.007\,t_1} \; e^{.005\,t_2}$	Mean $A_F/A_o = 1.30$
4 jet fires (n = 95)	$A_F/A_o = 1.16 \; e^{0.t_1} \; e^{.004 t_2} \; e^{.008 t_3}$	Mean $A_F/A_o = 1.25$
5 jet fires (n = 42)	$A_F/A_o = 1.03 \; e^{.07 t_1} \; e^{0 t_2} \; e^{.05 t_3} \; e^{0.t_4}$	Mean $A_F/A_o = 1.67$

All the R^2 values were very low and the regression coefficients were not significan or only significant at the 10% level. Most of the spread is accounted for by

the constant term. The results show only that spread increases with the delay in interarrival times and provides an order of magnitude estimate of the growth rates. This simple model (or at least this regression analysis) does not provide a reasonable basis for a model of fire growth.

4. A THEORETICAL MODEL OF FIRE GROWTH

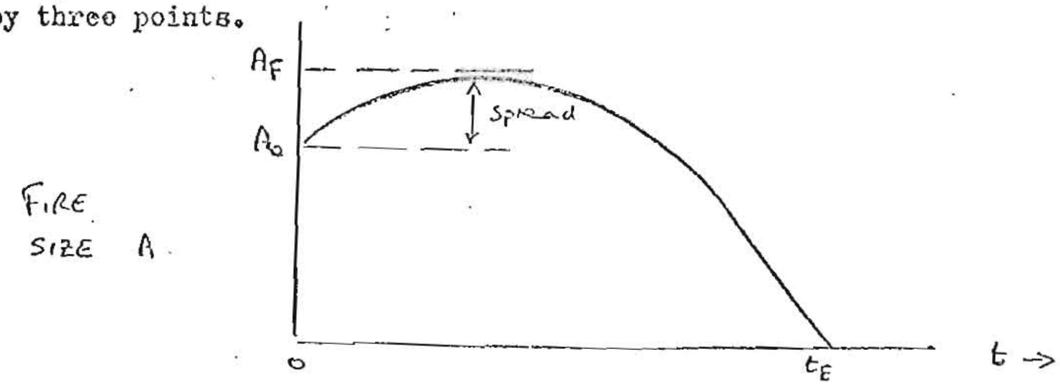
If the fire size is reduced by applying water to the perimeter of the fire, the time taken to extinguish the fire will be proportional to $A^{\frac{1}{2}}$ where A is the size of the fire, (See Baldwin, Melinck, Thomas . Fire Research Note No 884). If $\frac{d^2A}{dt^2} = -a$, the relationship between A to t_E (the extinction time) will be of the required form.

$$\frac{d^2A}{dt^2} = -a \qquad (4.1)$$

$$\frac{dA}{dt} = -at + b \qquad (4.2)$$

$$A = Ao - \frac{at^2}{2} + bt \qquad (4.3)$$

The only information we have about a fire is the area at arrival (Ao), the area damaged (A_F) and the extinction time (t_E). The information defines two points and a tangent on the graph of fire size against time. The quadratic equation 4.1, 4.2, 4.3 is the simplest function which will fit a curve defined by three points.



Equation 4.2 shows that the maximum area is reached at time t = b/a after arrival. This maximum fire size will be the area damaged at extinction.

$A_F = \text{Max}\left\{ A \right\} = Ao + b^2/_{2a}$ (4.4) and the absolute amount of spread is $b^2/_{2a}$

The extinction time is the time at which $A = 0$

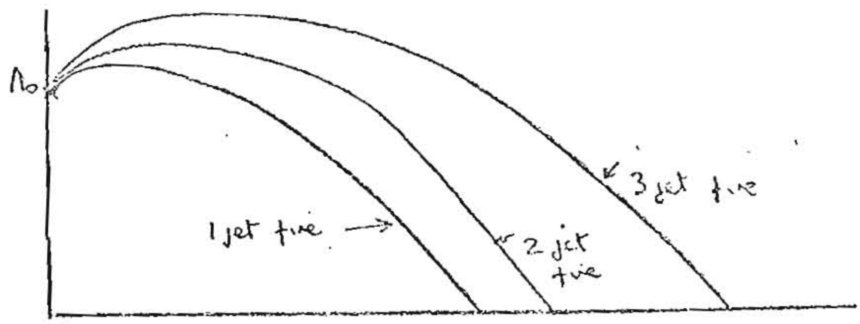
$$t_E = \frac{b \pm \sqrt{b^2 + 2aAo}}{a} = b/_a + \sqrt{\frac{2A_F}{a}} \qquad (4.5)$$

The parameters a and b can be given a physical interpretation. The parameter a determines how quickly the fire size is reduced. a will depend on the characteristics of the fire (size and "fierceness") and will also reflect the number of jets used to fight the fire. As more jets are applied the fire size will be reduced more quickly and the parameter a will increase.

The parameter b represents the continued growth of the fire as it is being fought. If the fire is not being fought a must have zero value and the fire growth will be $dA/_{dt} = b$. [Although b may not be constant, in which case $dA/_{dt} = b$ is a tangent to the growth curve at the point $A = Ao$]

Consider first the simplest situation where all the pumps which may be needed are available at the time of the first arrival and assume the parameters a and b are constants.

The fires will be characterised by their initial size Ao and their fierceness. The brigade will apply the appropriate number of jets according to their assessment of the fire size and severity. Fires of the same initial size. Ao will follow different growth curves according to their severity.



As the severity increases the amount of spread and control time also increase.

For a given Ao, as the severity increases the parameter a will decrease and/or the parameter b will increase.

It is assumed that all the differences in the growth of the fires are due to the differences between fires. We assume that the brigade apply the "right" number of jets, and do not allow for a "force" effect by suggesting that the fire may be put out more quickly by using more jets.

5. A METHOD OF ESTIMATING THE PARAMETERS FOR AN AVERAGE FIRE

We are still assuming that the parameters a and b are constants and considering only those fires for which all the pumps were available simultaneously. If we take a group of fires of the same initial size Ao, fought by the same number of jets, we can observe average values for the spread and control time from the SAF2 data. If we further assume that this group of fires has the same (or at least similar) a and b values then we can use the two observed averages to estimate the two parameters.

A rearrangement of the equations $A_F - Ao = b^2/2a$ and $t_E = b/a + \sqrt{2A_F/a}$ gives:

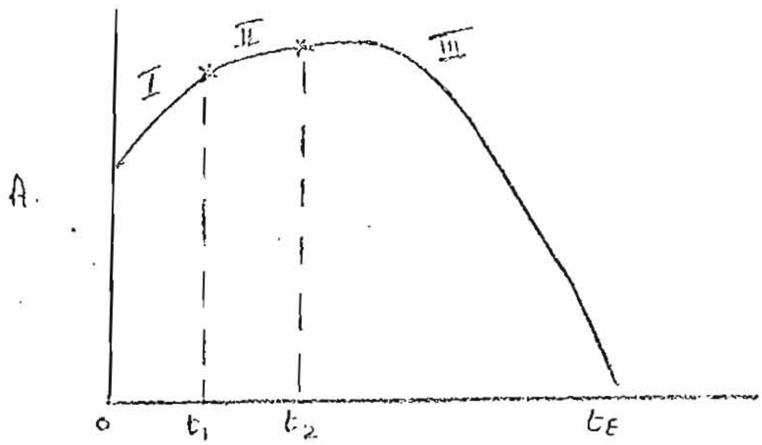
$$a = 2\left[\frac{\sqrt{k} + \sqrt{A_F}}{t_E}\right]^2 \qquad (5.1)$$

$$b = \sqrt{2ak} \qquad (5.2)$$

where k is the absolute amount of spread, $b^2/2a$

If we can make some assumption about the way in which 'a' changes as additional jets are brought into use, then we can generalise the growth equation to include those fires in which there was a delay in the arrival of the second and subsequent jets. In a study of wood crib fires (O'Dougherty and Young Fire Research Note No 603) it was found that the maximum amount of fire spread was proportional to (Rate of Water Application $)^{-1.24}$. If this result is used it would imply that $a_J = J^{1.24}\ a_1$ where a_J is the value of the a parameter when the fire is

being fought by J jets. In this preliminary analysis we have reasoned that in real fires the additional jets will have less effect than they would have in experimental fires and have used the simpler relationship $a_J = Ja_1$. If a fire is extinguished by J jets which arrive at $t_1, t_2, .. t_{J-1}$ after the first jet, the growth curve will be of the form shown below.



The equation of the growth in the final section will be

$$A = Ao + \int_o^{t_1} (b - a_1 t)\, dt + \int_{t_1}^{t_2} (b - a_2 t)\, dt + \dots + \int_{t_{J-1}}^{t} (b - a_J t)\, dt$$

$$A = Ao + bt + \frac{t_1^2}{2} (a_2 - a_1) + \frac{t_2^2}{2} (a_3 - a_2) + \dots - \frac{t^2}{2} a_3 \quad (5.3)$$

If $b/a_J > t_{J-1}$ the maximum will occur in the final section and the area damaged at extinction, A_F will be:

$$A_F = Ao + \frac{b^2}{2a_J} + \sum_{i=1}^{J-1} \frac{t_i^2}{2} (a_{i+1} - a_i) \quad (5.4)$$

The extinction time t_e is obtained from (5.3) by solving for t when A = O.

$$O = Ao + bt_E + \sum_{i=1}^{J-1} t_i^2/2 \, (a_{i+1} - a_i) - \frac{t_E^2}{2} a_J \quad (5.5)$$

If we use the relationship $a_i = i \times a_1$ then a_1 and b can be estimated from the simultaneous equations, (5.4) and (5.5)

6. ESTIMATES OF THE PARAMETERS OF THE CONSTANT QUADRATIC MODEL

The SAF2 sample of multi-storey non-dwelling fires which had spread beyond the room of origin were used to test the estimation methods. The fires were grouped according to the size at arrival, the number of jets used and the interarrival times if more than one jet was used. The interarrival times were calculated by assuming that i^{th} jet was applied at the time the i^{th} pump arrived. For each group of fires an average extinction time and average spread was calculated and the averages used to estimate a_1 and b as described in section 5. The results are shown in table 1.

Within the limitations of the data (see section 8) The results show:

(i) For a given Ao, a_1 decreases and b increases slightly with increasing number of jets.

(ii) for the two jet fires of a given Ao, the values of a_1 and b are approximately constant for various values of t_1.

(ii) a_1 and b both increase with increasing Ao.

Figures 1 and 2 show the relationship between a_1, b and Ao. The slopes of the log a_1/log Ao and log b/log Ao relationships are about the same and both a_1 and b are proportional to $A^{.75}$.

7. ALTERNATIVE FORMS OF THE QUADRATIC MODEL

A Model using Proportional Parameters

The parameters b and a represent the absolute rates of increase and decrease of the fire. It would seem reasonable to suggest that a and b might be functions of the fire size. If a and b are directly proportional to the fire size, then the model becomes:

$$\frac{dA}{dt} = -\alpha At + \beta A \qquad (7.1)$$

$$\log A = \log Ao - \frac{\alpha t^2}{2} + \beta t \qquad (7.2)$$

In the more general case where there are delays between the arrival of subsequent jets the spread will be:

$$\text{Log}\{A\ max\} - \log Ao = \log \frac{A\ max}{Ao} = \frac{\beta^2}{2\alpha} + \sum_{i=1}^{J-1} \frac{t_i^2}{2} (\alpha_{i+1} - \alpha_i) \quad (7.3)$$

and the control time can be estimated from the equation

$$0 = \log A + \beta t_E + \sum_{i=1}^{J-1} t_i^2/2 \, (\alpha_{i+1} - \alpha_i) - \frac{t_E^2}{2} \alpha_J \quad (7.4)$$

In this model the parameter β is the exponential growth factor. The estimates of α_1 and β are derived in an equivalent way to that described in section 6. The estimates are shown in table 2.

The estimates of α_1 and β are quite stable with respect to changing Ao. α_1 decreases with increasing number of jets but there is insufficient evidence to decide whether the growth rate β varies.

This model implies that the relationship between spread and arrival time is of the form:

$$\log A/Ao = \frac{\beta^2}{2\alpha_J} + \sum (\alpha_{i+1} - \alpha_i) \frac{t_i^2}{2}$$

This relationship can be tested by applying this form of the regression model to the SAF2 data. In the limited number of regressions which were timed it was found that this relationship gave a better fit, or at least no worse a fit, than the interarrival time model first tried (section 3).

An Intermediate Model

The relationship between the constant parameters a_1 b and Ao suggest a model in which a and b are proportional to $A^{.75}$.

$$\frac{dA}{dt} = b^1 A^{.75} - a^1 A^{.75} t \quad (7.5)$$

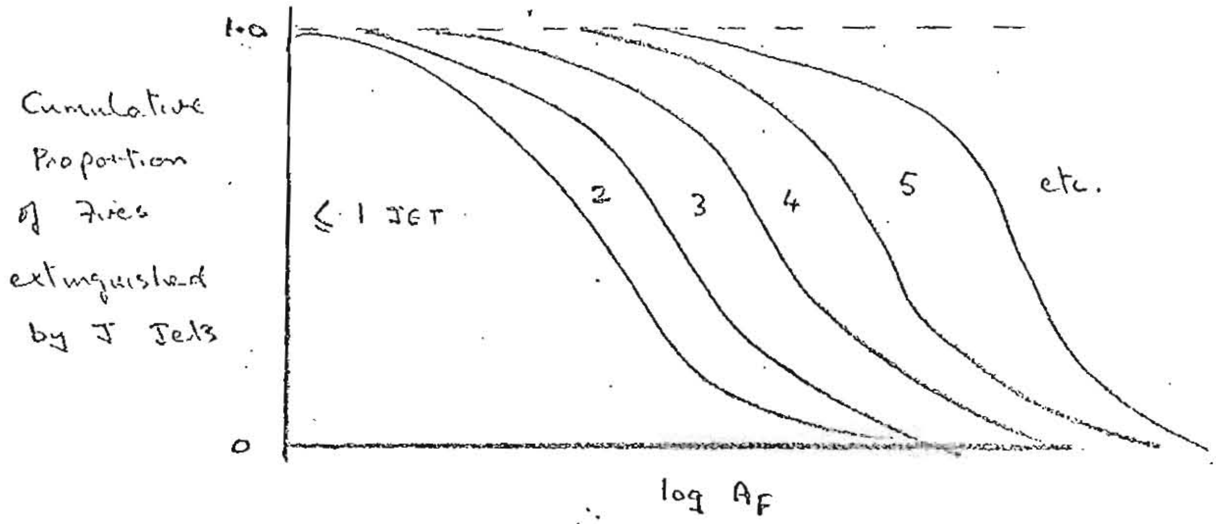
$$A^{\frac{1}{4}} = \frac{b^1 t}{4} - \frac{a^1}{8} t^2 + (A_o)^{\frac{1}{4}} \quad (7.6)$$

A more general model and an estimation procedure can be derived in a similar form to the other two models considered.

8. THE USE OF QUADRATIC MODEL

General Considerations - The amount by which a fire spreads after the arrival of the brigade depends on the characteristics of the fire and also on the action taken by the brigade. A model which is to be used to predict the expected growth of a fire must take into account the behaviour of both the fire and the brigade. The equations discussed in this note represent the fire growth for a given sequence of arrival times t_1, t_2 .... t_{J-1}. The sequence of arrival times will depend on the number of jets which are applied initially (from the first attendance or the assistance which is requested) and also on the subsequent decisions to bring on more jets as the fire grows.

The relationship between the area damaged at extinction and the total number of jets used has already been studied. The relationship can be expressed in the form shown below:



A fire of a given size can be characterised by its relative severity - ie a number between 0. and 100 which determines its position on the cumulative scale of numbers of jets required.

The following general principle could be used to ~~predict~~ simulate the growth of an individual fire which is of size Ao at arrival

i. Generate a random number to represent the relative severity of the fire.

ii. Apply the number of jets determined by the position of the fire on the jet area graph. These jets would be provided by the first attendance or, if necessary, assistance would be requested as soon as the first pump arrives.

iii. Determine the a and b parameters of the fire which will again be a function of area and relative severity.

iv. Predict the growth of the fire and check whether the fire at its maximum size and with the same severity index would require additional jets.

v. Request additional jets if required and continue extrapolating the growth of the fire with the changing a_i value.

9. LIMITATIONS OF THE DATA

The times at which the jets were applied - All we know is the arrival times of the pumps and we have had to assume some simple rule about the number of jets provided by each pump. For any particular fire we do not know how many jets came from each pump, or even whether the jets were applied immediately the pump arrived.

The Extinction Time - We have taken the 'extinction time to be the control time' as recorded on the K433 forms. The control time is the time between the arrival of the first pump and the "stop" message.

For the smaller fires the time of the "stop" message will approximate to the time the fire is "out", but for the larger fires the "stop" message may be sent well before the fire is "out".

The Amount of Spread - The amount of spread is taken from the area at arrival and final area damage as recorded on the SAF2 forms. The area at arrival will usually not be known with any accuracy, and is often recorded as being the same as the area at extinction. The areas are recorded to two significant figures

and this rounding error introduces additional inaccuracy.

Sample Variation The distributions of spread and control time are highly skewed and if one very large value is included in a sample this can completely outweigh all the other observations.

10. ALTERNATIVE METHODS OF ESTIMATING THE PARAMETERS

1. Estimate the parameters for each homogeneous group of fires. Each group consists of fires which have the same area at arrival, number of jets and interarrival times. Obtain an array of parameters for each jet-area group by interpolation and smoothing of the group estimates.

2. Estimate the parameter values for each individual fire, possibly introducing a random element designed to "put back" the rounding error. Determine the parameters for each jet-area group by taking the average of the individual values in the group. The reason for "putting back" the rounding error is to get a continuous distribution of spread and hence a continuous distribution of estimates of a and b.

3. Obtain a regression relationship between spread and interarrival times, and extinction time and interarrival times. Use the regression to estimate the spread and extinction time for fires of a given area and number of jets. Estimate the a, b parameters for this group using the regression estimates of spread and control time.

4. Use the regression relationship between spread, control time, number of jets and area at arrival to estimate a, b directly by considering the correspondence between the coefficients of the regression model and the quadratic model.

11. FURTHER WORK

The Relationship between a Deterministic and a Stochastic Model

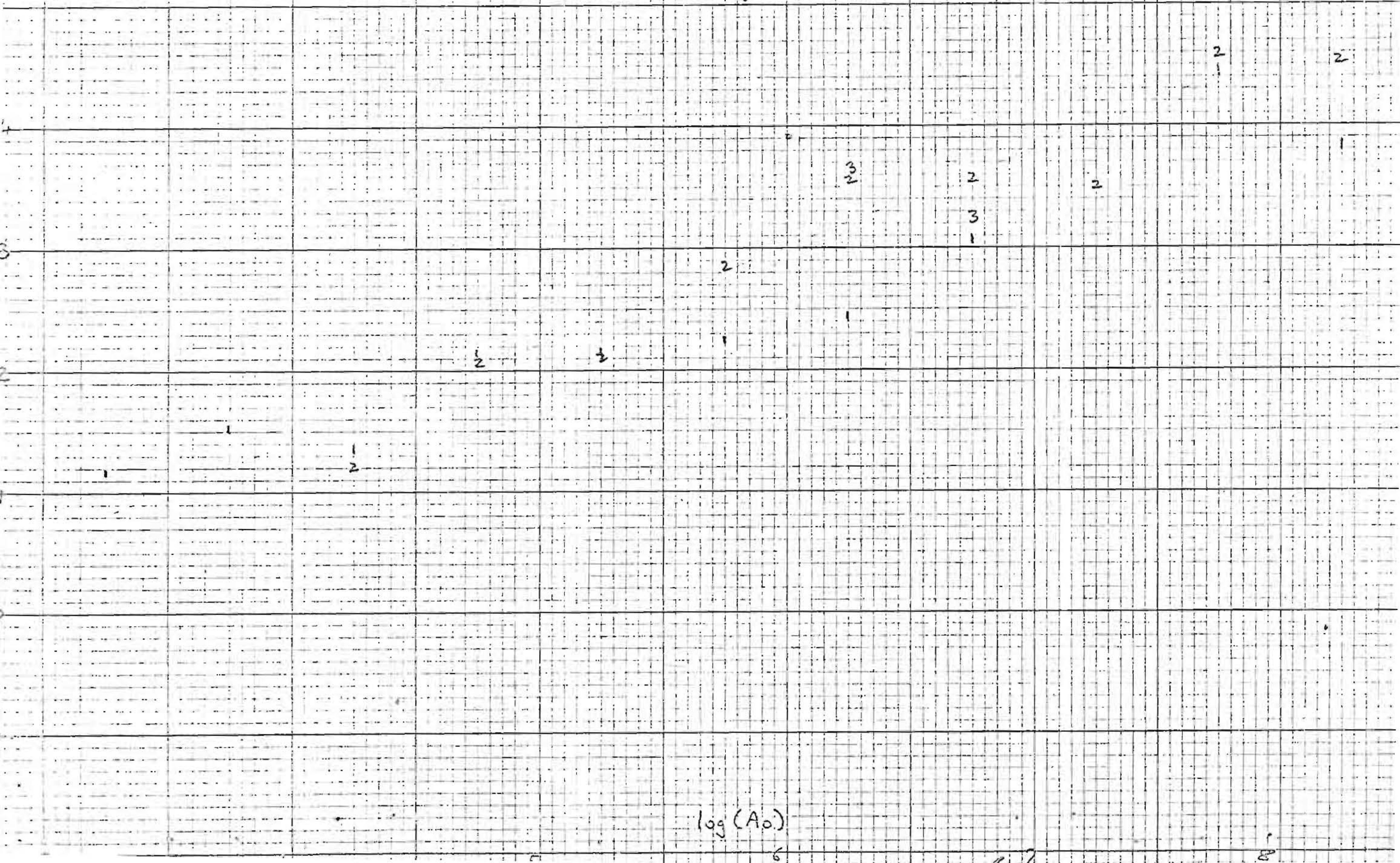
We have assumed that the parameters a and b, are constant for each group of fires, whereas in reality there will be a distribution of a and b values. The

behaviour of a fire with average a, b values may not be the same as the average of all fires with individual a, b values. The properties of the stochastic model need to be examined, and in particular it may be worthwhile considering a model in which some fires have zero b values (no growth) and the remaining fires have positive b values.

Validation It should be possible to validate (or to disprove) this proposed model by a more thorough study of the regression relationships obtainable from the SAF2 data.

Restricted Fire Growth The models considered here are assumed to have no restrictions on growth. In practice many fires will be confined by the building or at least some fire compartment within the building. The effect of building size must be examined.

Fig. 1 The relationship between b and A_o for 1, 2 and 3 jet fires – MSND out of room fires



for the quadratic constant

Area at Annual (ft^2) (log groups)	1 JET FIRES		2 JET FIRES $t_1 = 0$		$t_1 = 1$		$t_1 = 2$		$t_1 = 3$	
	a_1	b	a_1	b	a_1	b	a_1	b	a_1	b
27	1.03	3.2	.2	2.1						
42	.56	4.6	.14	.3						
71	.77	3.9	.3	3.5			.15	3.1		
113	1.32	8.5	.6	8.2						
173	1.61	8.7	1.1	11.2			.7	5.3	.9	11.[illegible]
322	2.85	9.3	.9	12.1	.8	19.0			2.7	2[illegible]
543	1.71	11.8	2.9	34.8	1.07	12.15			2.8	62.[illegible]
829	5.36	22.4	4.4	34.4			8.1	55.8		
1440	28.3	—	3.4	38.0			2.8	22.8	1.7	3[illegible]
1975	10.1	84.2	15.7	—	9.05	95.9				
3750	9.5	47.1	7.6	93.5						

parameter model.

3 JET FIRES

$t_1 = 5$		All 2 JET FIRES		$t_1 = 0$, $t_2 = 0$		$t_1 = 0$, $t_2 = 1$		$t_1 = 0$, $t_2 = 2$		$t_1 = 0$, $t_2 = 5$	
a_1	b	a_1	b	a_1	b	a_1	b	a_1	b	a_1	b
		·2	2·1								
		·3	3·5								
		·6	8·2								
·22	3·1	·75	9·0								
1·1	29·5	1·1	18·0								
1·8	25·3	2·5	33·2	2·8	51	·8	17·7				
1·6	34·7	4·1	36·9	·9	31·0	1·9	36·9	1·1	20·0	·7	21·5
2·7	34·6	2·9	34·2							1·0	25·8
3·0	113·0	11·4	103	2·9	—						
		7·6	93·5							4·0	148

TABLE 2 Estimates of α_1 and β for out of some [illegible] fires for the quadratic proportional parameter model.

Area at arrival (ft^2)	1 set fires		2 set fires ($b_1 = 0$)		3 set fires	
	α_1	β	α_1	β	α_1	β
27	·085	·169	·018	·10		
42	·026	·139	·010	·05		
71	·032	·089	·012	·08		
113	·032	·119	·014	·10		
173	·032	·089	·019	·10		
327	·040	·061	·011	·08		
543	·014	·045	·020	·12	·016	·15
829	·032	·060	·024	·08	·004	·063
1440	·053	–	·012	·06		
1975	·023	·086	·012	–	·009	–
3750	·017	·032	·012	·06		

FIG. 2 The relationship between a_i and A_o for 1, 2, 3 diet fires – MSNJ out of room fires.